presents difficulties not to be treated with levity. To make a selection of commodities, agricultural and manufactured, to be used as the standard, to assign to each its coefficient of relative importance, to determine the quantity of money and bullion in the country, and to ascertain the amount of the transactions settled by offset, are problems which will probably be regarded more seriously by the practical financier than by the ingenious theorist. By one difficulty, however, M. Walras finds himself pressed. As a certain fluctuation of prices with the times must in any case be expected and allowed for, can the future movements of price be foretold accurately enough, to insure against an issue of token money on the eve of a rise of prices from other causes, or a withdrawal when prices would have fallen? If this cannot be insured against, the plan in operation might even exaggerate the fluctuations which it is desired to prevent. This and similar difficulties of administration it seems hard to escape, if our dependence has still to rest on the range of view and foresight of merely human agencies.

THE ARITHMETIC, GEOMETRIC, AND HARMONIC MEANS.

The late Mr. Jevons repeatedly declared his preference for the geometric mean in economic investigations instead of the arithmetic mean, commonly used. He has also referred to the possibility of employing the harmonic mean. But he has nowhere left an explicit statement of his reasons for preferring one mean to the other.

The possibility of choice between different means does not exist in most of the cases which the student of physical science encounters. The mean is generally for the physicist the manner in which he infers the magnitude of a real quantity from a number of varying measurements. As the mean represents a real quantity, it can evidently have but one true value, whether our defective knowledge enables us to know which mean gives this or not. Such a mean, representing a real quantity, is known according to its nature as the Precise Mean Result or as the Probable Mean Result.

Thus, the physicist makes several measurements of the length of a strip of steel. These measurements differ slightly from unavoidable errors in adjusting and reading the instrument; but the error is equally likely to fall on either side of the truth, and the true length is most probably the arithmetic mean of the measurements. For the proof of this proposition generally received, see Chauvenet's *Astronomy*, vol. ii., Appendix. Such a mean, where the causes cannot be eliminated with perfect certainty, is called the Probable Mean Result. Again, the physicist may wish to obtain the exact weight of a body. No balance is exactly poised so that the two arms are precisely equal. But their difference effects the result by a known law of mechanics. This law shows that by weighing the body first in one pan and then in the other, and taking the geometric mean of the two weights, we shall get the exact answer. Such a mean is the Precise Mean Result.

The mean commonly employed by the economist is of a quite different character. It is not a real quantity at all, but is a quantity assumed as the representative of a number of others differing from it more or less. It is made to stand for these other real quantities, for convenience in reasoning about them or in drawing general conclusions from them collectively. This is the Fictitious Mean or average, properly so called. Its fictitious character renders it possible to make choice among different values, and thus among different methods of finding it. This is generally overlooked by those who invariably use the arithmetic mean as if it were the only one which could be applicable. The only justification for any fictitious mean is to be found in its convenience as a representative of the true quantities. It is upon this criterion that Mr. Jevons based his choice.

The fictitious mean necessarily errs from the correct value for each real quantity which it represents. Mr. Jevons seems to imply that the mean to be chosen is that which most fairly divides the error among the quantities.

He considers three means, the arithmetic, the geometric, and the harmonic. The arithmetic mean is that commonly used.

It may be defined to be the quotient of the sum of the quantities by their number. Its general formula is:—

$$x = \frac{a + b + \ldots . m}{n}$$

Its great simplicity recommends it for general use, and it at first sight seems to distribute the error very fairly. Thus, if we are considering the change in price of two commodities, one of which has remained unchanged, while the other has doubled in price, we take the arithmetic mean of 100 per cent. and 200 per cent., and get the average price, 150 per cent., a rise of 50 per cent. This is an error of 50 of the units employed for each quantity. But an error of 50 in estimating a quantity of 100 units is much more serious than in a quantity of 200 units. As we are at liberty to choose our mean as we like, however, we may still take the arithmetic mean, when we think it fairer to favor the commodity which has risen most. This may be right, when that commodity is much the more important. The arithmetic mean will always be found to give a smaller proportional error for the larger quantities.

The geometric mean, which Mr. Jevons prefers, is defined as the *nth* root of the continued product of n quantities. It may be defined in terms of the arithmetic mean as the quantity whose logarithm is the arithmetic mean of the logarithms of the given quantities. Its formula would then be:—

$$\log x = \frac{\log a + \log b + \ldots . \log m}{n}$$

The use of logarithms makes the geometric mean almost as convenient as the arithmetic. There are cases, such as the determination of the population at any time, its rate of increase being known, in which the economist must employ this mean. The mean is here a real quantity, and not fictitious. Mr. Jevons also thinks that it should be used in averaging changes of prices, and the like. There seems to be no reason why this mean should be used in such cases, except the very insufficient one that its result will fall between that of the arithmetic and that of the harmonic. Mr. Jevons seems

to imply that the error is more fairly distributed, and points out that, in the case above supposed, the geometric mean, about 141 per cent., would be more just, because, as he remarks, $\frac{100}{141} = \frac{141}{200}$. But there seems to be no reason why the fact that the ratio of the smaller quantity to the average is equal to the ratio of the average to the larger quantity should be considered of importance. If, as seems desirable, we seek to apportion the error fairly among equally important quantities, we must take the harmonic mean.

The harmonic mean, defined in terms of the arithmetic, is the quantity whose reciprocal is the arithmetic mean of the reciprocals of the given quantities. Its formula is:—

$$\frac{1}{x} = \frac{\frac{1}{a} + \frac{1}{b} + \cdots\cdots \frac{1}{m}}{n}$$

This is a very awkward mean to calculate, which renders it undesirable for general use. But it does give the error fairly according to the size of the quantities. Thus, in the supposed case, the average price would be about 133 per cent., an error of 33 or one-third for the smaller quantity, an error of 67 or one-third for the larger.

The geometric mean may be better for general use, because it combines simplicity with an approach to a fair distribution. The fact that the mean is a fictitious quantity, and that it should be so taken that the error may be least misleading, is the ultimate criterion, and may determine a different choice in different cases. See Jevons's *Investigations in Currency and Finance*, pp. 23 and 120, and the same author's *Principles of Science*, 3d ed., chap. xvi., for the discussion upon which this note is based. F. COGGESHALL.

LEGISLATION FOR LABOR ARBITRATION.

On account of the increased number of lock-outs and strikes during the past year, the legislatures of several of the States holding sessions for 1885–86 felt it their duty to make some provision for boards or tribunals of arbitration created or sanctioned by the State. Four States — Iowa and Kansas in the